

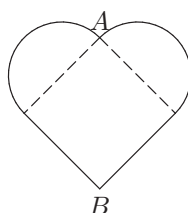
GEOMETRY SUPERHEROES - CIRCLES AND POLYGONS

Time: 60 minutes

Directions: Each of the 30 multiple-choice questions is followed by five possible answer choices. Choices A through D will provide answers, while Choice E is for none of the above. Scoring will be as follows: 5 points for a correct answer, 0 for a wrong answer, and 1 for a blank answer (for a maximum possible score of 150). Figures are not necessarily drawn to scale. *Do not choose incomplete answers: if a question has more than one solution, do not select a choice that only provides one of the solutions!*

Calculators are not permitted.

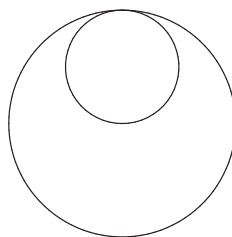
1. What is the maximum number of points at which two different circles can intersect?
(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA
2. The exterior angle of a regular polygon has measure 1° . How many sides does the polygon have?
(A) 36 (B) 180 (C) 360 (D) 1800 (E) NOTA
3. P and Q are on a circle centered at O such that $\angle POQ = 120^\circ$. If circle O has radius 6, what is the length of \widehat{PQ} ?
(A) 2π (B) 4π (C) 6π (D) 12π (E) NOTA
4. A heart is made from a square and two semicircles whose diameters are sides of the square. In the given diagram, A and B are opposite vertices of the square, and the length of AB is 20. Find the area of the heart.



- (A) $100 + 25\pi$ (B) $200 + 25\pi$ (C) $200 + 50\pi$ (D) $200 + 100\pi$ (E) NOTA
5. Distinct points A , B , and C lie on a circle such that AC is a diameter of the circle. What is the measure of $\angle ABC$?
(A) 45° (B) 60° (C) 90° (D) 180° (E) NOTA
 6. Given three points in a plane, it is always possible to draw a circle that passes through all three?
(A) Yes (B) No (C) $1 = 2$ (D) Love bugs were made at UF. (E) NOTA

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7. $A_1A_2A_3 \dots A_{20}$ is a regular 20-gon inscribed in a circle centered at O . Find the measure of $\angle A_1OA_2$.
- (A) 9° (B) 10° (C) 20° (D) 30° (E) NOTA
8. A 6×8 rectangle is inscribed in a circle. What is the area of the circle?
- (A) 10π (B) 25π (C) 48π (D) 100π (E) NOTA
9. An equilateral triangle has an inscribed circle of radius 3. Find the side length of the triangle.
- (A) 3 (B) $3\sqrt{3}$ (C) 6 (D) $6\sqrt{3}$ (E) NOTA
10. Points A , B , and C are on a circle such that $\angle ABC = 40^\circ$. What is the measure of \widehat{ABC} ?
- (A) 40° (B) 80° (C) 280° (D) 320° (E) NOTA
11. A target consists of three concentric circles with radii 1, 2, and 3. If a dart is thrown at random onto the target, what is the probability it lands inside the middle circle but outside the small circle?
- (A) $1/9$ (B) $1/3$ (C) $4/9$ (D) $5/9$ (E) NOTA
12. Two circles are internally tangent so that the diameter of one circle is a radius of the other. If the area of the larger circle is A , what is the area inside the larger circle but outside the smaller circle?



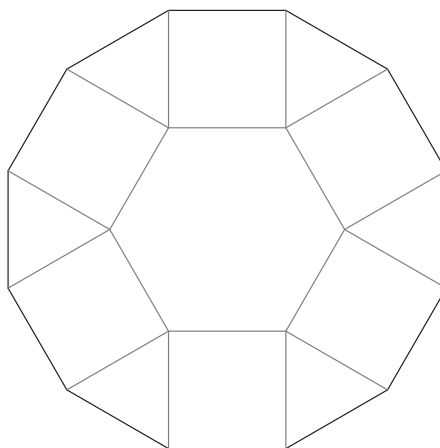
- (A) $\frac{1}{4}A$ (B) $\frac{1}{2}A$ (C) $\frac{2}{3}A$ (D) $\frac{3}{4}A$ (E) NOTA
13. A rhombus has side length 8, and one of its angles measures 120° . What is the sum of the lengths of the diagonals of the rhombus?
- (A) $4 + 4\sqrt{3}$ (B) 16 (C) 20 (D) $8 + 8\sqrt{3}$ (E) NOTA
14. Quadrilateral $ABCD$ is inscribed in a circle. If $\angle ABC = x$, find the measure of $\angle CDA$.
- (A) $x/2$ (B) x (C) $90^\circ - x$ (D) $180^\circ - x$ (E) NOTA

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15. Quadrilateral $ABCD$ is inscribed in a circle. If $\angle ABD = y$, find the measure of $\angle ACD$.

(A) $y/2$ (B) y (C) $90^\circ - y$ (D) $180^\circ - y$ (E) NOTA

16. A regular dodecagon can be dissected into six equilateral triangles, six squares, and one regular hexagon as shown. What is the area of a regular dodecagon with side length 1?



(A) $3 + 3\sqrt{3}$ (B) $6 + 3\sqrt{3}$ (C) $3 + 6\sqrt{3}$ (D) $6 + 6\sqrt{3}$ (E) NOTA

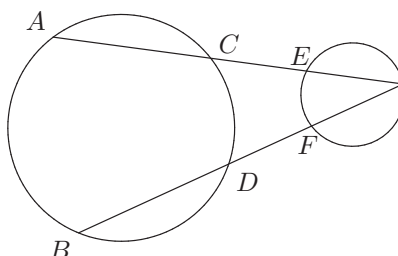
17. Circles P and Q have radii 4 and 3, respectively, and are externally tangent to each other. A line passing through point P is tangent to circle Q at E . Find PE .

(A) 5 (B) $2\sqrt{10}$ (C) $\sqrt{58}$ (D) 7 (E) NOTA

18. Points P , Q , R , and S lie on a circle such that $\overline{PQ} \parallel \overline{RS}$. If $\widehat{PR} = 10^\circ$, find \widehat{QS} .

(A) 20° (B) 30° (C) 40° (D) 50° (E) NOTA

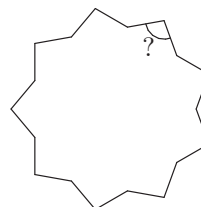
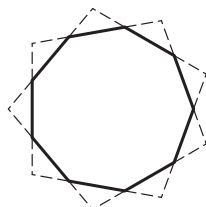
19. Two circles are intersected by two lines as shown. If $\widehat{AB} = 120^\circ$ and $\widehat{EF} = 70^\circ$, what is the measure of \widehat{CD} ?



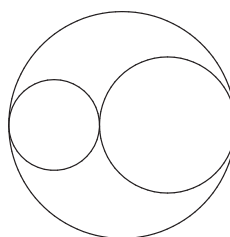
(A) 25° (B) 50° (C) 85° (D) 95° (E) NOTA

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20. Triangle ABC is inscribed in a circle with center O . If $\angle ABC = 40^\circ$, what is the measure of $\angle OAC$?
- (A) 35° (B) 40° (C) 50° (D) 80° (E) NOTA
21. Rogue has two squares with areas 6 and 10. She cuts up these two squares into pieces, then reassembles the pieces into a single square (so that all the pieces are used). She then takes the side length of the new square, and the side length of each of the original squares, and forms a triangle with these three side lengths. What is the area of this triangle?
- (A) $\sqrt{15}$ (B) 4 (C) 8 (D) 16 (E) NOTA
22. The sides of a regular nonagon are extended to form an 18-sided star, as shown below. Find the angle measure of a point of the star.



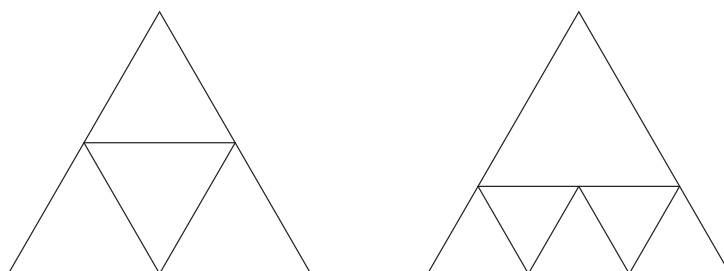
- (A) 100° (B) 120° (C) 140° (D) 160° (E) NOTA
23. Let $A_1A_2A_3 \dots A_{12}$ be a convex dodecagon. Two distinct diagonals *cross* if they intersect at a point in the interior of the dodecagon. How many diagonals cross diagonal A_1A_5 ?
- (A) 21 (B) 24 (C) 27 (D) 32 (E) NOTA
24. Two circles are externally tangent to each other and internally tangent to a larger circle, as shown. The centers of the two smaller circles lie on a diameter of the largest circle. If the area inside the largest circle but outside the smaller circles is 64π , what is the product of the radii of the two smaller circles?



- (A) 8 (B) 16 (C) 32 (D) 64 (E) NOTA

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25. Three distinct, congruent circles of radius 6 all pass through a point P . Find the radius of the circle passing through the centers of these three circles.
- (A) $2\sqrt{3}$ (B) 6 (C) 9 (D) $6\sqrt{3}$ (E) NOTA
26. In convex hexagon $ABCDEF$, $AB = DE$, $BC = EF$, $CD = FA$, $\overline{AB} \parallel \overline{DE}$, $\overline{BC} \parallel \overline{EF}$, and $\overline{CD} \parallel \overline{FA}$. If $AC = 5$, $BD = 6$, and $CE = 8$, find the perimeter of triangle BDF .
- (A) 13 (B) 18 (C) 19 (D) 19.5 (E) NOTA
27. A *lattice point* is a point in the coordinate plane with integer coordinates. Wolverine creates a convex polygon in the coordinate plane by drawing line segments between lattice points that are either 1 unit or $\sqrt{2}$ units apart. What is the maximum number of sides this convex polygon can have?
- (A) 4 (B) 6 (C) 8 (D) 12 (E) NOTA
28. Bobby has an equilateral triangle, and he wants to dissect it into n smaller equilateral triangles. For $n = 4$ and $n = 6$, this is possible, as shown below. For which of the following values of n is it also possible: 7, 8, and 9?



- (A) 8 only (B) 7 and 8 only (C) 7 and 9 only (D) 7, 8, and 9 (E) NOTA
29. Let \mathcal{P} be a convex polygon with n sides and perimeter p . Region \mathcal{Q} is the set of all points outside of \mathcal{P} that are at most 1 unit away from a side or vertex of \mathcal{P} . Find the area of \mathcal{Q} .
- (A) $p + \pi$ (B) $p + \frac{n-2}{2}\pi$ (C) $p + (n-2)\pi$ (D) $p + n\pi$ (E) NOTA
30. $ABCDEF$ is a convex equilateral hexagon with $\angle ABC = \angle AFE = 100^\circ$ and $\angle FAB = 140^\circ$. Find the measure of $\angle CDE$.
- (A) 100° (B) 120° (C) 140° (D) 160° (E) NOTA