1. The first 17 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59. Since 59 is the 17th smallest prime number, *x* – 2 = 59 – 2 = 57 **(C)**

2. This is a right triangle with side lengths of 6 and 8 miles and a hypotenuse of 10 miles, since. Therefore, the area of this triangle is = 24 **(B)**

3. In order for there to be no gaps or overlaps in the pattern, the particular polygon must be able to tessellate. In order for a polygon to tessellate, each interior angle measure for the polygon must be a divisor of 360**°.** With each interior angle having a measure of 108**°**, a regular pentagon would not fit this description. **(C)**

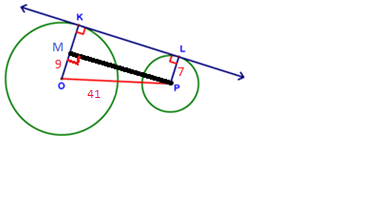
4. The distance between these two points is . Since the question asked for the square of the distance, the answer is 122 **(A)**

5. The complement of the supplement of Angle J (in degrees) is equal to , which we can set equal to , as given in the question. Solving this, we get

, which simplifies into , so . Since Angle J equals , plugging in x to this expression gives us 110**°** **(D)**

6. Let the longest side of the triangle be *c*, and let the other two sides by *a* and *b*. The triangle will be acute only if . Since , the answer with side lengths that makes an acute triangle is 5, 11, 12 **(B)**

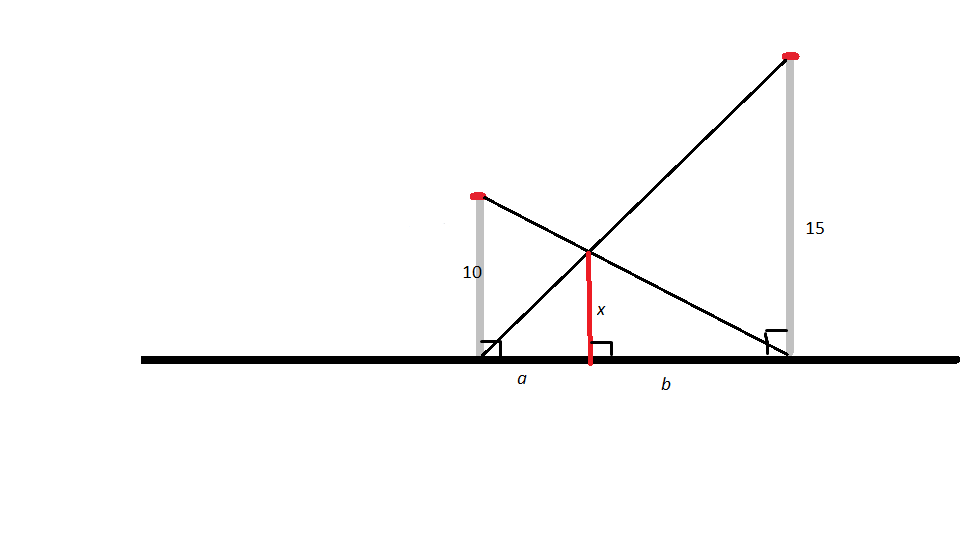
7. The common external tangent is given by the line segment between points *K* and *L* in the diagram below. The centers are 41 units apart, so *OP* = 41. Since the common external tangent is tangent to both circles, the radii of both circles form right angles with the tangent where they intersect. We can make a rectangle *KLPM* with a width of *MK* = *LP* = 7 (radius of planet is 7) and a length of *MP* = *KL*, which is what we are trying to find. Since OK = 16, OM = OK – MK = 16 – 7 = 9. ∠*OMP* is a right angle, so △*OMP* is a right triangle. Using the Pythagorean Theorem, , so MP = , and therefore KL = . **(E)**

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8. The distance from a point (m, n) to the line Ax + By + C = 0 is equal to . Rearranging , we get , so after plugging everything into the equation, we get . **(A)**

9. The area of this footprint is the sum of the area of a rectangle (with side lengths 4 and 7) and the area of two semicircles. The semicircles on either end have radii of 2, since the diameter is 4. Thus, the area of the footprint is . **(C)**

10. In this problem, we are trying to find the value of *x* in the picture below. We can see that , due to the similarity of triangles. We can also find that due to similarity of triangles again. When we add these two equations together, we get . Therefore, , so 6. **(B)**

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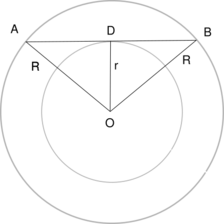
11. According to definition, both incenters and centroids must be within the triangle, so the answer is I & II. **(B)**

12. The formula to find the volume of a conical frustum is , where *h* is the height of the frustum, *R* is the radius of the base, and *r* is the radius of the top of the frustum. Using the numbers from the question, we get = (**D)**

13. The sum of the interior angles of a polygon is given by , where *n* is the number of sides of the polygon. A hexagon has 6 sides, so the sum of the interior angles is 180(6 – 2) = 720**°**. Since the smallest angle has a measure of 105**°**, the other angles are and , with *d* being the common difference of the arithmetic sequence. Adding all 6 angles together, we get (sum of interior angles), so . Therefore, the largest angle of the hexagon is 135. **(C)**

14. Since △*ABC* is a right triangle, we can use the Pythagorean Theorem to find that , so . In order to get back in time, Joanna and Jennifer must travel these 3900 meters in 5 minutes, or 300 seconds, so . **(A)**

15. If we draw the picture below, where point *O* is the center of both concentric circles and *R* is the radius of the outer circle, we see that the tangent chord *AB* forms a right angle with the radius of the black hole (inner circle), denoted as *r*. It can also be seen that point *D* is the midpoint of the tangent chord *AB*, so *AD* = *DB* = 7. Using the fact that △*ODB* is a right triangle, we see that , so . In order to find the area in between the inner and outer circles, we must notice that this equals . **(C)**



16. If the original statement is denoted as “If p, then q”, the converse of this statement is “If q, then p”. Therefore, the converse of the statement “If I am sucked into a black hole, then I will die” is “If I will die, then I am sucked into a black hole”. **(C)**

17. As the number of sides in a regular polygon increases, the polygon begins to resemble a circle more and more. Thus, we can approximate the perimeter of a 2000-sided regular polygon by finding the circumference of a circle with the same radius as the polygon. The circumference of a circle with radius is **(B)**

18. The number of diagonals in an n-sided polygon equals , so the number of diagonals in a 200-sided polygon is **(A)**

19. If a triangle has coordinates and , the centroid of the triangle is . Therefore, the centroid of the triangle with coordinates and is . **(D)**

20. Using the Shoelace Theorem for finding the area of a polygon, we put the coordinates in lines, as shown to the right, repeating the first coordinate at the end. Then, we multiply the numbers that are connected with the “shoelaces”, and we add up the totals we get on each side. Then, we take the absolute value of the difference between -8 and 22 and divide this by two, giving us . **(C)**

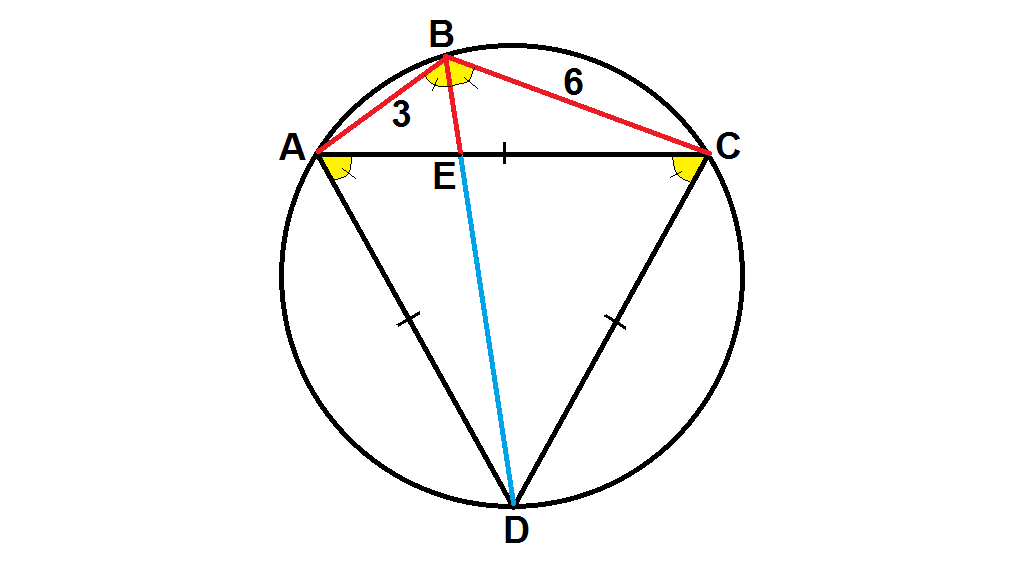
3 1 7 7 -6 -18 -30 5 5 35 +15 3 1 + 5

-8 22

21. Since the nebula is in the shape of a cyclic quadrilateral, we can apply Ptolemy’s Theorem, which states that Plugging in the given values to the equation, we get . Simplifying this equation, we get  **(C)**

22. Using the picture of cyclic quadrilateral *ABCD* below, △*ACD* is equilateral (since all side lengths are equal). Therefore, ∠*CAD,* ∠*CDA,* and∠*DCA* are all 60**°**. Since ∠*ABD* has the same intercepted arc as ∠*DCA*, these angles have the same measure. Since ∠*CBD* has the same intercepted arc as ∠*CAD*, these angles also have the same measure. Now we can see that the measure of ∠*CBD=*∠*ABD*, and so *BD* is an angle bisector.

Applying the Angle Bisector Theorem to △*ABC*, . Letting *EC* = , we get , and solving this equation, we get , so , and **(E)**.



23. The Triangle Inequality Theorem states that the sum of any 2 sides of a triangle must be greater than the measure of the third side. Therefore, the third side of this triangle cannot have a length of 17 (which is 5 + 12) or greater, and it also cannot have a length of 7 (which is 12 – 5) or lower. The only integer values that satisfy the Triangle Inequality Theorem in this case are: 8, 9, 10, 11, 12, 13, 14, 15, and 16, so the third side length could be 9 different possible values. **(B)**

24. Since every edge of the hexagonal prism has a length of 3, the bases of the prism are regular hexagons with side length 3. The area of a hexagon with side length *a* is , so the area of each base is . After accounting for the area of both bases, which is , we must find the surface area of the 6 faces connecting the bases. The surface area of these 6 faces is of . To get the total surface area of the hexagonal prism, we add to get  **(B)**

25. Rhombuses, squares, and rectangles are all parallelograms, and one characteristic of parallelograms is that the diagonals bisect each other. However, this is not necessarily true for the trapezoid. **(D)**

26. The indentation that would be made by the asteroid is a hemisphere (half of a sphere), so the surface area = . Since the diameter is 90 miles, the radius (or *r*) is 45 miles. Plugging this value into the formula, we get  **(A)**

27. If a secant and a tangent of a circle are drawn from a point outside the circle, then the product of the lengths of the secant and its external segment equals the square of the length of the tangent segment. In this case, . Therefore, , so . Letting *x* = *PB*, we get , which factors to , and so , but since length cannot be negative, is extraneous. *PB* = *x* = 7. **(C)**

28. The diameter of the circle is also the length of the diagonal of the inscribed square, so the diagonal of the square equals twice the radius of the circle, or . The diagonal of the square is also the hypotenuse of a 45-45-90 triangle, with the other sides being the sides of the square. Since the ratio of the sides in a 45-45-90 triangle is 1:1:, the side length of the square is . The area of the square is = 8. **(D)**

29. This triangle has a height of 9 and a base length of 4. After rotating the triangle around the y-axis, a cone with a height of 9 and a radius of 4 is formed. The volume of a cone with radius *r* and height *h* is , so the volume of the cone formed here is . **(E)**

30. The length of the median of a trapezoid is the average of the two base lengths. Therefore, the median *EF* = 17.5 = , so . . **(B)**