**Now that Joanna and Jennifer have successfully completed their tour of our solar system, it’s time for part two of their space adventure: the unknown. You’d better buckle your seatbelts, because Joanna and Jennifer are headed for the deepest, darkest, most mysterious parts of our universe, in which they will encounter all sorts of scary space stuff. Get ready for one thrilling space story!**

**Choose the letter of the correct answer. In all cases, E) NOTA means “none of these answers”.**

1. The first stop on part two of Joanna and Jennifer’s space adventure involves a trip to a singing black hole! The lowest sound wave from an object in the universe ever detected by humans was a B-flat that came from a black hole in a group of galaxies called the Perseus Cluster. This particular B-flat is *x* - 2 octaves below middle C, where *x* is the 17th smallest prime number (with 2 being the 1st smallest). How many octaves below middle C was the B-flat from the black hole?

**A) 45 B) 51 C) 57 D) 59 E) NOTA**

2. In order to reach their first stop, Joanna and Jennifer must take a shortcut by going through a wormhole. If the entrance to the wormhole is a triangle with sides of 6 miles, 8 miles, and 10 miles, what is the area of this entrance, in square miles?

**A) 12 B) 24 C) 36 D) 48 E) NOTA**

3. After entering the wormhole, Jennifer notices that the inside “walls” of the wormhole contain a pattern made entirely up of a single regular polygon, with no gaps or overlap. Which of the following regular polygons could Jennifer NOT have seen inside the wormhole?

**A) Triangle B) Square C) Pentagon**

**D) Hexagon E) NOTA**

4. The wormhole takes Joanna and Jennifer from the point (3, 4, 5) to the point (10, 7, -3). What is the square of the distance between these two points?

**A) 122 B) 124 C) 126 D) 128 E) NOTA**

5. If Angle J has a measure of degrees, and the complement of the supplement of Angle J has a measure of degrees, what is the measure of Angle J, in degrees?

**A) 20 B) 25 C) 90 D) 110 E) NOTA**

6. When Joanna and Jennifer exit the wormhole, they are surprised to find that they took the wrong wormhole and are actually nowhere near the singing black hole! Instead, they see a peculiar new planet, which they decide to check out. Which of the following could represent the side lengths of an acute triangle?

**A) 3, 4, 5 B) 5, 11, 12 C) 6, 7, 11 D) 8, 10, 14 E) NOTA**

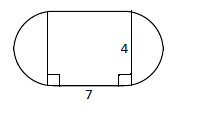
7. The new planet of radius 7 units is orbiting a larger black hole with radius 16 units. The centers of the planet and the black hole are a constant 41 units apart. Assuming that both the planet and the black hole can be represented as 2D circles with the given radii, what is the length of the common external tangent between the planet and the black hole, in units?

**A) B) C) D) E) NOTA**

8. Joanna and Jennifer, out of curiosity, decide to further investigate this planet, which they name Planet J, by actually landing on the surface. In order to do so, the girls need to figure out their distance from the planet. If the path of their spaceship can be modeled by the line , and the planet can be modeled by the point (10, 3), what is the shortest distance between their path and the planet?

**A) B) C) D) E) NOTA**

9. Once they land, Jennifer is the first to get off the spaceship. Her boot makes a footprint in the shape of a rectangle with two semicircles on either side. What is the area of the footprint, as shown below, if the rectangle has a length of 7 and a width of 4?



**A) B) C) D) E) NOTA**

10. In order to commemorate their arrival, Jennifer decides to put up a flag on a 10-foot pole, while Joanna puts up a flag on a 15-foot pole that is 20 feet away from Jennifer’s flagpole. If the girls used two strings to connect the top of each flagpole to the other flagpole’s base, how high above the ground would the two strings intersect?

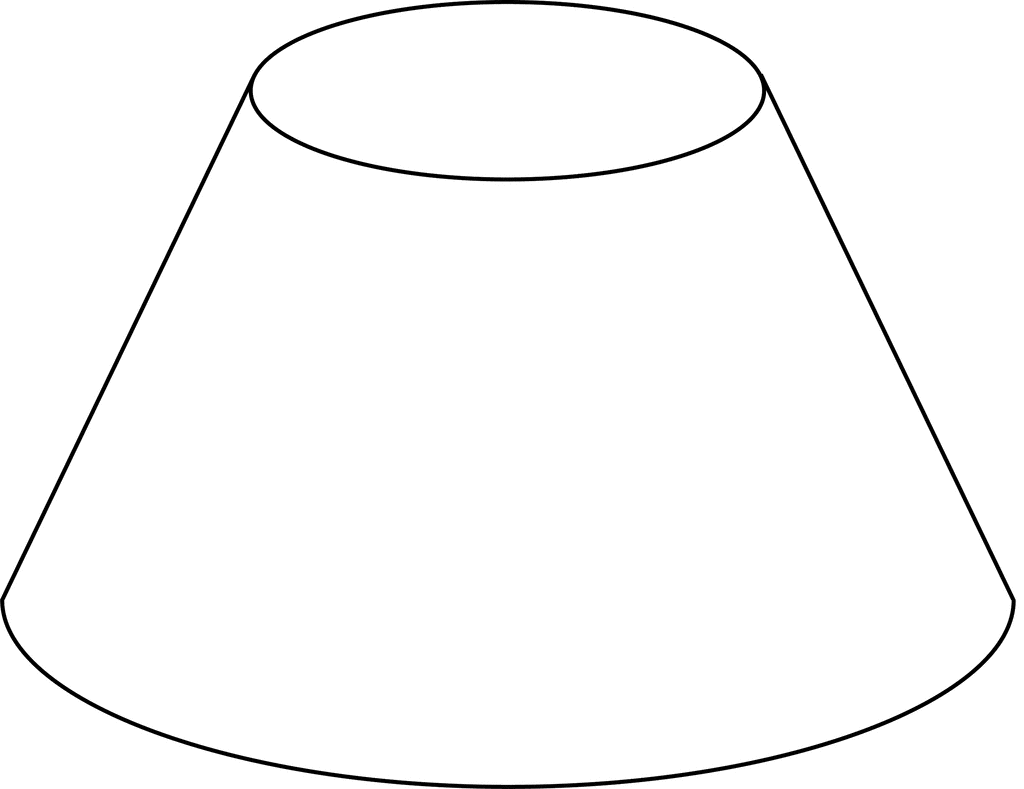
**A) 8 B) 6 C) 4 D) 2 E) NOTA**

11. Which of the following will always be on the interior of a triangle?

I. Centroid II. Incenter III. Circumcenter IV. Orthocenter

**A) II only B) I & II C) II & III D) I, II, & III E) NOTA**

12. While exploring Planet J, Joanna and Jennifer find a large volcano in the shape of a conical frustum, or a cone with the top part cut off, as seen in the picture below. If the radius at the base of the volcano is 10, the radius of the hole at the top of the volcano is 4, and the height of the frustum is 6, what is the volume of the volcano?

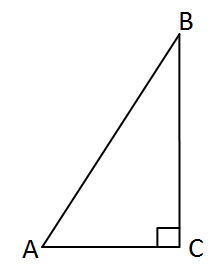


**A) B) C) D) E) NOTA**

13. If the interior angles of a particular hexagon form an arithmetic sequence such that the smallest angle has a measure of 105°, what is the measure of the largest angle of the hexagon, in degrees?

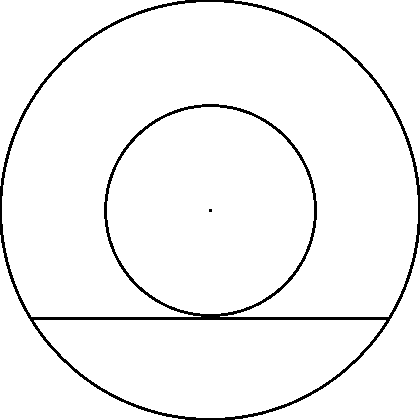
**A) B) C) D) E) NOTA**

14. Joanna and Jennifer, who are at point *A*, feel a strange rumbling beneath their feet and realize that the volcano, at point C, is about to erupt! They have exactly 5 minutes to get back to their spaceship at point *B* before the volcano erupts. If the girls are 1500 meters from the volcano, while their spaceship is 3600 meters from the volcano, what is the minimum speed (Joanna hates running) in meters per second, they must both travel in order to reach their spaceship before the volcano erupts, assuming that they take the shortest route to their spaceship?



**A) B) C) D) E) NOTA**

15. After barely escaping the volcano eruption on Planet J, the girls head over to the black hole that Planet J is orbiting. The black hole’s event horizon is the boundary in which nothing can escape the black hole’s pull. In the picture below, the event horizon is the outer circle, while the black hole is represented by the inner circle. If the chord shown below is tangent to the inner circle and has a length of 14, what is the area in between the inner circle and the outer circle?



14

**A) B) C) D) E) NOTA**

16. Joanna thinks of the statement “If I am sucked into a black hole, then I will die.” Which of the following statements is the converse of the statement in the previous sentence?

**A) “If I am sucked into a black hole, then I will die.”**

**B) “If I am not sucked into a black hole, then I won’t die.”**

**C) “If I will die, then I am sucked into a black hole.”**

**D) “If I do not die, then I am not sucked into a black hole.”**

**E) NOTA**

17. Whoops! Jennifer has accidently pressed the acceleration on their spaceship, so now the two girls are actually heading straight for the entrance of the black hole! If the entrance of the black hole is a regular polygon with 2000 sides and a radius of , which of the following is closest to the perimeter of the black hole’s entrance?

**A) B) C) D) E) NOTA**

18. Luckily, Joanna is able to think quickly, and she presses the button to initiate the brakes on the spaceship before the girls get sucked into the black hole. However, the spaceship requires that Joanna type in the number of diagonals of the black hole’s entrance in order for the brakes to work. Quick, can you help Joanna figure out what number to type in, assuming that the black hole entrance is a polygon with 200 sides?

**A) B) C) D) E) NOTA**

19. After too many near-death experiences on this adventure, Joanna and Jennifer decide that it is time to return back to Earth. What is the centroid of the triangle with the coordinates and ?

**A) B) C) D) E) NOTA**

20. What is the area of the triangle from question #19?

**A) B) C) D) E) NOTA**

**Use this information to answer questions 21-22:**

A nebula is an interstellar cloud of dust, hydrogen, helium and other ionized gases. A particular nebula in space that our dynamic duo passes on their trip back to Earth is in the shape of a cyclic quadrilateral with vertices A, B, C, and D such that *AB = 3*, *BC = 6*, and *AC = AD = CD =* . Let the point of intersection of *AC* and *BD* be *E*.

21. What is the length of *BD?*

**A) B) C) D) E) NOTA**

22. What is the length of *AE*?

**A) B) C) 3 D) E) NOTA**

23. If a triangle has two sides with lengths of 5 and 12, how many different possible values could the third side length be, given that the third side must have an integer length?

**A) B) C) D) E) NOTA**

24. What is the surface area of a hexagonal prism with all edges having a length of 3?

**A) B) C) D) E) NOTA**

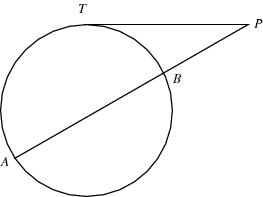
25. All of the following types of quadrilaterals have diagonals that bisect each other except:

**A) Rectangle B) Rhombus C) Square D) Trapezoid E) NOTA**

26. Oh no! An asteroid is heading towards Earth and will leave a hemispherical indentation in Earth’s surface if Joanna and Jennifer are not able to stop it. If the indentation would have a diameter of 90 miles, what is the surface area of the indentation that would be left behind by this meteor, in cubic miles?

**A) B) C) D) E) NOTA**

27. If the circle in the picture below represents Earth and point *P* represents the asteroid, what is the length of *PB*, given that *AB* = 13 and TP = ?



**A) 5 B) C) D) E) NOTA**

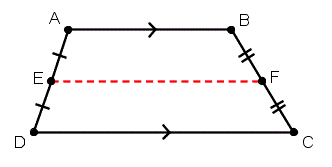
28. On Joanna and Jennifer’s spaceship, there is a button that they can press to shoot a freeze ray at the asteroid, which will stop it from hitting Earth. The button is in the shape of a circle with a square inscribed in it. If the radius of the circular button is 2 inches, what is the area of the inscribed square, in square inches?

**A) B) C) D) E) NOTA**

29. What is the volume of the figure created when the triangle with coordinates and is rotated around the y-axis?

**A) B) C) D) E) NOTA**

30. Yay! Joanna and Jennifer were able to use their freeze ray and stop the asteroid from hitting the Earth! Because they’re complete nerds, the two girls decide to celebrate by doing a math problem: If *EF* (as shown below) is the median of the trapezoid *ABCD*, what is the length of *AB*, given that *EF* = 17.5 and *CD* = 22?



**A) B) C) D) E) NOTA**