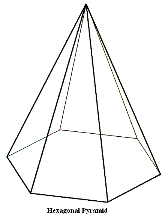
**You have 60 minutes to complete the following 30 multiple-choice questions. Choices A through D are answer choices for every problem. Choice E stands for “none of these answers,” or NOTA. Scoring is as follows: 5 points for a correct answer, 1 point if left unanswered, and 0 points for an incorrect response. Units are assumed. Diagrams not to scale.**

Jennifer and Joanna are about to embark on an amazing space adventure! Of course any trip must start close to home, so they are going to first take a tour of our solar system. They will explore the eight (no matter your thoughts Pluto is not a full-fledged planet on this test!) planets revolving around the sun. But what is an adventure without a little risk? Soon Jennifer and Joanna will be leaving the comfort of their solar system and venture into much darker realms of the universe. That’s for later though. For now enjoy the comfort of your own solar system. Have fun!

1. Before they can even think about adventuring, Jennifer and Joanna must design a rocket ship that can take them. Joanna suggests the shape of the rocket ship to be a hexagonal prism with a base side length of eight feet and a height of forty feet. What is the surface area of the prism?

**A. B. C. D. E. NOTA**

2. Brandon yells at Joanna for such a bad design because “it is not very aerodynamic.” He says it should be a regular pyramid instead with the same height and base side length as Joanna’s suggestion. What is the volume of Brandon’s pyramid?

**A. B. C. D. E. NOTA**

3. Joanna, Jennifer, and the newly constructed rocket are collinear with Jennifer being in the middle. Joanna is standing one foot away from Jennifer and Jennifer is standing 1660 feet from the rocket, but because Joanna is shorter than Jennifer, she can’t see the rocket. If Joanna is 64 inches tall and Jennifer is 68 inches tall, what is the greatest possible height (in feet) of the rocket? Assume Joanna’s eye level is equivalent to her height.

**A. 550 B. 553 C. 557 D. 559 E. NOTA**

4. It’s finally time for take-off. Before entering space, Jennifer and Joanna must go through the layers of Earth’s atmosphere (troposphere, stratosphere, mesosphere, thermosphere, exosphere). The layers represented by the horizontal lines are parallel to ones another and the transverse marks the straight trajectory of their flight. What are angles 1 and 2 called?

1

2

**A. Vertical angles**

**B. Corresponding angles**

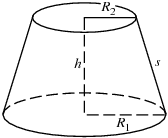
**C. Alternate exterior angles**

**D. Alternate interior angles**

**E. NOTA**

5. The rocket that the pair is riding on has three thrusters on the bottom, each in the shape of a frustum of a cone with top radius 5 yards, bottom radius 12 yards, and a height of 10 yards. If each thruster is capable of producing 3,000 pounds of thrust per cubic foot of volume, what is the max thrust capability in pounds of all three thrusters combined?

frustum



**A. 2,290,000 B. 20,610,000 C. 61,830,000**

**D. 185,490,000 E. NOTA**

6. Two hours after taking off, Jennifer decides that she’s bored. Joanna loves logic so she proposes a game. “I’ll give you a statement and you tell me the contrapositive! If Joanna is cool, then she has friends.” What should Jennifer respond with?

**A. If Joanna is not cool, then she does not have friends.**

**B. If Joanna does not have friends, then she is not cool.**

**C. If Joanna has friends, then she is cool.**

**D. If Joanna is not cool, then she has friends.**

**E. NOTA**

7. After playing this game for a while, Jennifer is bored again (she just doesn’t appreciate logic)! So Joanna gives her a harder question this time. “Tell me the original statement if the converse of the inverse of the contrapositive of the converse is: If the game is boring, then tell me this.”

**A. If the game is boring, then tell me this.**

**B. If you tell me this, then the game is boring.**

**C. If you don’t tell me this, then the game is not boring.**

**D. If the game is not boring, then don’t tell me this.**

**E. NOTA**

For questions 8-11 assume the solar system can be modeled by a point in the center as the sun and eight concentric circles representing the orbits of each of the planets. Mercury’s orbit has a radius of 1 and each succeeding orbit has a radius increased by 1 unit.

8. How far away is Mars from Neptune when they are collinear with the sun between them?

**A. 4 B. 7 C. 8 D. 10 E. NOTA**

9. Jennifer and Joanna are on Venus’s orbit and have been alone for a while now. Jennifer is tired of Joanna and wants to visit her other friends. Luckily Brandon and DZ are enjoying themselves together on another ship also on Venus’s orbit! In order to get to them, Jennifer and Joanna travel in a straight line tangent to Mercury’s orbit. What distance must they travel?

**A. B. 2 C. D. 4 E. NOTA**

10. Once they arrive at Brandon and DZ’s ship, Jennifer and Joanna go inside. DZ, with his undying love of coordinate geometry must have a Cartesian plane with him at all times. Thus, a coordinate grid was built into the floor of their spaceship. Joanna notices that the four of them standing on the grid form a parallelogram. Brandon is at , Joanna is at , Jennifer is at , and DZ is at . After a half an hour of DZ explaining the beauty of geometry, Jennifer decides she has had enough and wishes to move **farther** away from him. However, the perfection of the parallelogram is not unnoticed by Jennifer so she makes sure that the parallelogram is maintained at her new spot. What are the coordinates of her new position?

**A. B. C. D. E. NOTA**

11. It’s finally time for them to depart. They both have other parts of the solar system to explore. The two rockets ships leave Venus’s orbit at an angle of 15° and then both intersect Mercury’s orbit twice (shown right) and at the same time as each other. At the initial intersection with Mercury’s orbit, the minor arc between them measures 30°. What is the distance between the two ships when the cross Mercury’s orbit again?

Venus

Mercury

**A. 1 B. C. D. 2 E. NOTA**

12. Unbeknownst to Brandon and DZ, Joanna took with her a square picture of the sun and Mercury with side length 2 feet. The sun, due to its large size is not entirely on the picture. With radius 1 foot, its center is on the lower left corner. Mercury’s size pales in comparison, but just for fun, Joanna wants to find the maximum size Mercury could be on the picture without it overlapping with the sun or going off the square picture. What is the radius of the largest possible Mercury that could fit in the picture?

**A. B. C. D. E. NOTA**

13. After Brandon’s ship and Joanna’s ship leave each other, DZ drops Brandon off on Earth, the only planet with Brandon’s favorite fruit, the succulent apple. After landing on the planet, Brandon does the only logical thing he can—eat apples. He first takes one bite, then one-half of a bite, then one-fourth of a bite, then one-eighth of a bite, and so on. If there is an infinite amount of apples for Brandon to eat on the planet, how many bites did he take total?

**A. 2                B.            C. 4               D. an infinite number of bites   E. NOTA**

14. After passing Mars, Joanna and Jennifer were already bored beyond their wits. They decided to play a game they made up, called schlops. They have two fair dice, numbered one to six, and they each take turns rolling the dice, starting with Joanna. If the numbers on the dice Joanna or Jennifer rolls add up to a prime number, they win. However, if the sum is not prime, the other person gets to roll the dice, until one of the two rolls a prime number sum and wins. What is the probability that Jennifer wins their game of schlops?

**A. B. C. D. E. NOTA**

15. Jennifer and Joanna are finally in the depths of despair (asteroid belt). This is the most dangerous part of their solar system venture because they must navigate between the many asteroids. However, this is Joanna’s favorite part because she loves looking at the different three dimensional shapes! The first notable asteroid they pass is an icosahedron. What is the sum of the number of faces, vertices, and edges of this asteroid?

**A. 2         B. 20        C. 42   D. 52 E. NOTA**

16. Jennifer squeals in excitement because she sees a small asteroid hurl by on the other side of the ship in the shape of a regular tetrahedron. Joanna rushes over and estimates the side length to be about 2 feet long. Joanna knows the approximate density of the asteroid and quickly calculates the mass (using her estimate of the edge length) to be grams. Which density did Joanna use to calculate the mass of the asteroid? ()

**A. 32 g/in3  B. 36 g/in3  C. 44 g/in3  D. 48 g/in3  E. NOTA**

17. Jennifer notices three asteroids around them. Their coordinates are, , and . What is the perimeter of the triangle formed but these asteroids?

**A. 26 B. C. D. 42 E. NOTA**

18. Then, suddenly, an asteroid smashes into a wing of their spaceship and broke it into millions of indiscernible pieces, catapulting through the vacuum of space. Luckily Brandon built an auto-repair mechanism into the ship. Unluckily, Brandon (wanting them to die) put a lock on the button. In order to begin auto-repair, they must enter “the product of the number of faces of the all the platonic solids.” What code should they enter to begin auto-repair?

**A. 2800 B. 11,040 C. 38,400 D. 46,080 E. NOTA**

19. After the accident, Joanna begins pondering things that crash/intersect with each other, in particular, points of concurrency. Which of the following points of concurrency must always be inside the triangle?

I. Centroid II. Orthocenter III. Circumcenter IV. Incenter

**A. I only B. I and III C. I, and IV D. I, III, and IV E. NOTA**

20. Jennifer and Joanna are anxiously waiting for the rocket to repair, but alas the ship is good as new again. And excellent timing because they are quickly approaching Jupiter, the largest planet in the solar system. Joanna’s extensive research comes in handy again, for she knows of the multiple layers of Jupiter. In the center is a rocky core, and the next surrounding layer is metallic hydrogen. The outermost layer is molecular hydrogen. To the side is a cross section of Jupiter. About what percentage of Jupiter’s volume is not metallic hydrogen?

20,000 km 40,000 km

20,000 km

**A. 36% B. 41% C. 50% D. 51% E.NOTA**

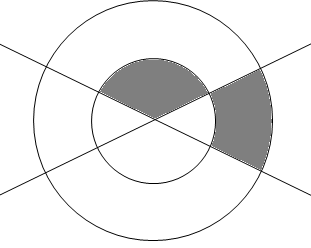
21. In desperate need of a break from the monotony of the spaceship, the pair decide to stop at one of Jupiter’s moons, Io. Jennifer spots a mountain that she wishes to climb, though they must go up the less steep side because Joanna is very out of shape. They find that the mountain can be modeled by a triangle with a base of and the lengths of the two sloping sides are 82 and . What is the angle of elevation of the side of the mountain they will climb up?

**A. 20° B. 30° C. 40° D. 45° E. NOTA**

22. Once reaching Saturn, the pair is in awe of all the magnificent rings of the planet. However, this awe does not last long, as the nerds immediately burst into super math mode! Since neither is very adept at art, they each draw a picture of Saturn as two concentric circles. On Jennifer’s diagram, starting on a point from the outer circle, a straight line segment tangent to the inner circle is drawn so that it touches another point on the outer circle. If this line has length 20 cm, what is the area between the two circles?

**A. 20π cm2   B. 100π cm2 C. 400π cm2 D. not enough information E. NOTA**

23. Joanna HATES her drawing of Saturn in which the concentric circles have radii of 4 and 6 inches, so she draws a giant X through the center. She notices that the two shaded regions bounded by the X are of equal area. What is the angle marked α?



α°

**A. 80° B. 100° C. 110° D. 120° E. NOTA**

24. Jennifer begins to miss her dear friends Brandon and DZ, so Joanna decides to contact them. However, she cannot because there is something directly in the way of the signal. It is Earth! Luckily, Joanna has a contact on Earth, named Jessica, who can send Brandon and DZ a message at a low cost of $0.000015 per mile the message needs to travel from Earth to get to Brandon. The ratio of the distance from Brandon’s ship to Earth to the distance from Jennifer’s ship to Earth is 2:5. The distance between their two ships is 511,000,000 miles. How much money must Joanna pay Jessica for her message?

**A. $2190 B. $2986 C. $3066 D. $3308 E. NOTA**

25. While flying by Jupiter, the pair also notices an abundance of moons, 63 to be exact. How many triangles can be formed by connecting the moons? Assume no three moons are collinear.

**A. 21 B. 63 C. 39,711 D. 238,266 E. NOTA**

26. In the distance, Jennifer spots a large blue-green mass. Uranus, of course! Uranus has 13 known rings. Which of the following is a name of a 13-gon?

**A. triskaidecagon B. tredecagon C. tricerodecagon D. tridecigon E. NOTA**

27. Uranus has an axial tilt of 98°. If 98° is the included angle for two sides of a triangle of length 3 and 4, which of the following could be the length of the third side?

**A. 3.3365 B. 4.7845 C. 4.9035 D. 5.3235 E. NOTA**

28. Neptune has an orbital radius of about 30 astronomical units (AU). One AU defined as the distance from Earth to the sun. Assuming their orbital paths to be circular, what is the ratio of the circumference of Earth’s orbit to the circumference of Neptune’s orbit?

**A. 1:30 B. 1:300 C. 1:900 D. 1:2700 E. NOTA**

29. Soon their journey beyond the solar system is nearing, but for now they pass Pluto. Jennifer sees Pluto through one of the rocket’s triangular windows. Her perspective is just right so that Pluto looks like a circle inscribed in the triangular window. It appears that Pluto has a radius of 1 foot and Jennifer knows perimeter of the rocket’s windows is 12 feet. What is the area of the window not taken up by Pluto?

**A. B. C. D. cannot be determined E. NOTA**

30. How many planets (not exoplanets or dwarf planets) are in the solar system?

**A. 7 B. 8 C. 9 D. 10 E. NOTA**