✔️1.      You can write 592 - 412 as (59 + 41)(59 - 41) because it is a difference of squares. This is equivalent to (100)(18) = 1800. Alternatively, you can write square both numbers. 592 - 412 = 3481 - 1681 = 1800, **D.**

✔️2.     The number 76 has 2 prime factors, 2 and 19. So, any number that has a factor of 2 or 19 will not be relatively with 76. We can count the number of numbers not relatively prime with 76 and then subtract the number we get from 75. There are 37 even numbers less than 76 and 3 numbers less than 76 divisible by 19. However, we double-counted the numbers less than 75 that are divisible by both 2 and 19. There is one number less than 76 that is divisible by both 2 and 19. So, there are 37+3-1=39 numbers less than 76 that are not relatively prime to 76. Now, computing 75-39, we get 36. **C**

✔️3.      If we set the number of cars he should buy as x, then we can make the equation x = 3x - 8. Solving for x gives us x = 4. **D**

✔️4.      The first Avenger can shake hands with the other 6 Avengers. The next Avenger can only shake hands with 5 other Avengers, because he/she already shook hands with the first Avenger (and they only shake hands once). The next Avenger can only shake hands with 4 other Avengers and so on. 6 + 5 + 4 + 3 + 2 + 1 = 21, **E**.

✔️5.      On the sides 30 miles long, 16 stakes can be placed on each side. On the sides 20 miles long, 11 stakes can be placed on each side. The corner stakes are each double counted, so we must subtract 4. Now, we just solve 16 + 16 + 11 + 11 - 4 = 50, **B**.

✔️6.      f(x) = 3x4 - 4x2 + 2x - 6 so f(-2) = 3(-2)4 - 4(-2)2 + 2(-2) - 6 = 48 - 16 - 4 - 6 = 22, **C**.

✔️7.     Using some guessing and checking we can see that the side lengths are 7, 9, and 12. Then, the volume of this rectangular prism is 7 \* 9 \* 12 = 756, **C**.

✔️8.      We can use the distance formula. Plugging in the given coordinates, we get √(6 - 1)2 + (-6 - 6)2 = √169 = 13, **C**.

✔️9.      The slope of Vision’s path is ⅔, the slope of War Machine’s path is ⅔, and the slope of Falcon’s path is -3/2. Summing these 3 values together, we get -⅙, **B**.

✔️10.  One train is traveling at 400 miles per hour and the other train is traveling at 350 miles per hour. This means that in one hour, together the trains cover a distance of 750 miles. Because they started 6000 miles apart, it will take 6000/750 hours before the trains crash together, which is 8 hours. Iron Man is flying at a speed of 50 miles per hour. Because he can change direction instantly and never stops flying, he will be able to fly 50 \* 8 miles before the trains crash, which is 400 miles, **C**.

✔️11.  The factors of 48 in order are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. The sum of the factors is 1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 + 48 = 124, **C**.

✔️12.  You can find the area of each ring by subtracting the area of the inner circle from the outer circle. The area of the inner gray ring is the area of the gray circle minus the area of the black circle. This is equal to 36π - 9π = 27π. The area of the outer gray ring is the area of the entire circle minus the area of the white circle. This is equal to 144π - 81π = 63π. The sum of both gray rings is 27π + 63π = 90π, **E**.

✔️13.  The product of 2 numbers is equal to the product of their GCF and LCM. So, we simply do 15 \* 225 = 3375, **E**.

✔️14.  The numbers -20 and 100 are 120 units away from each other. The smallest positive prime number, 2, and the largest positive prime number, 97, are 95 units away from each other. So, we get 95/120 which simplifies to 19/24[, **A**.](https://www.codecogs.com/eqnedit.php?latex=%5Cfrac%7B19%7D%7B24%7D%250)

✔️15.  There are 3 non-number cards (face cards) in each suite and 4 suites in a standard 52-card deck, making a total of 3 \* 4 = 12 face cards. Therefore, the probability of Hulk not drawing a number card is (52 - 40)/52 = 12/52.. There are 13 diamonds in a deck. Therefore, the probability of Clint drawing a diamond is 13/52. However we already counted the 3 diamond face cards so that is only an additional 10 chances. Therefore the probability is 12+10 cards out of 52 = 22/52 = 11/26 **B**.

✔️16.  If a 300 mL mixture is 99% vibranium, it contains 297 mL of vibranium, so it must contain 3 mL of non-vibranium. The non-vibranium will remain unchanged at 3 mL, because we will only remove the vibranium, as given in the problem. Therefore, if we want to get the mixture to be 97% vibranium, or 3% non-vibranium, we must have 100 mL of total solution, because 3/100 = 3%. If we want to have 100 mL of total solution, then this is the same as having 97 mL of vibranium. Now we solve the starting amount of vibranium minus the desired amount of vibranium remaining which equals  297 - 97 = 200 mL, **D**.

✔️17.  Using the formula d = rt, we can see that Vera runs at a speed of 10 = 5r ➝ r = 2 mph. The total distance of the two total laps is 10 + 10 = 20 miles, at an average speed of 5 mph. Using the formula, 20 = 5t ➝ t = 4 hours. However, it took Vera 5 hours to run her lap which is already more than the 4 hours it would take for Vera to run one lap and Kaitlin to run one lap. Therefore, there is no possible answer,  **E**.

✔️18.  Easy solution: We can use plug and chug to solve this system of equations. We know that at least one of the variables must be negative because they must add up to 3, and their squares have to add up to 29. If they were all positive, the sum of the squares would be too small. We find that the three numbers are -3, 4, and 2. Because *a ≤ b ≤ c,* a = -3, b = 2, c = 4. Therefore, *6a + 7b + 5c* = 6(-3) + 7(2) + 5(4) = -18 + 14 + 20 = 16, **C**.

More rigorous solution:

We can find the values of  a + b + c, ab + bc+ ca, and abc, and then write a cubic polynomial to solve for a, b, and c. a + b + c is already given as 3. We can square the first equation, subtract the second equation, then divide by 2 to solve for ab + bc + ca. Doing so, we get ab + bc+ ca = -10. To solve for abc, we can use the identity a3 + b3 + c3 - 3abc = (a + b + c) (a2 + b2 + c2 - (ab + bc + ca)). Plugging in our known values, we get 45 - 3abc = 3 \* (29 - (-10)). Simplifying the equations gives abc = -24. Now, using Vieta’s Formulas, we can write the cubic polynomial x3 - 3x2 - 10x + 24 = 0. The roots of this polynomial are -3, 2, and 4. These are also the values of a, b, and c. We know that a ≤ b ≤ c, so a = -3, b = 2, and c = 4. We can now compute 6a + 7b + 5c = 16, **C**.

✔️19.  The sum of the numbers in the 1st group is 1. The sum of the numbers in the 2nd group is 3 + 5 = 8. The sum of the numbers in the 3rd group is 7 + 9 + 11 = 27 and so on. We can see that the sum of the numbers in the nth group is just n3. So, to find the sum of the numbers in the 6th and 7th group, we just do 63 + 73 = 559, **D**.

✔️20.  0.5555… can be expressed as 5/11, so it is not an irrational number. Real numbers are any numbers without i, so 5/11 is a real number. Integers are …-3, -2, -1, 0, 1, 2, 3, … so 5/11 is not an integer. Rational numbers are numbers that can be expressed as a fraction, so 5/11 is a rational number. Therefore, 0.5555… real and rational or II and IV, **D**.

✔️21. The stock is originally worth $2100. An increase of 1100% means that the stock increased 11 times as much. 11 \* 2100 = 23100. An increase of $23100 means that the new total price of the stock is $23100 + $2100 = $25200, **C**.

✔️22.  There are 9 letters in the word TESSERACT. You can rearrange the letters 9! times. However, we must account for the repeats from multiple letters. The letter T, E, and S are all repeated twice. This gives us a result of 9!/(2!2!2!) which is equal to 45360, **A**.

✔️23.  The first class has 19 scores so the median is a number in the data set but the lower or upper quartile is not a number in the data set. So there are 9 numbers between 79 and 90, 4 in each quarter and the median. The second data set has 21 numbers so the median is a data point in the data set but the upper and lower quartile are not. The median is 80 and there are 5 numbers in the 3rd quarter for 6 numbers in the second class. Total of 15 scores that are between 80 and 89. 15/40 = 37.5% **C**

✔️24.  We can work backwards for this problem. We can start with the ending value, 0, and add 20 to see how much money Thor would have had before he paid Heimdall the 5th time. We find that Thor had $20 after crossing the bridge 4 times. Then we can halve the value and again add 20, repeating it 4 times because Thor crossed the bridge 4 times. First we start with $20, then get $30, then get $35, then get $37.50, and finally end with $38.75 as our answer, **C**.

✔️25.   Store 1 sells 17 capes for $15, so the cost of one cape is 15/17 = .882 = 88 cents. Store 2 sells 35 capes for $31, so the cost of one cape is 31/35 = .886 = 89 cents. Store 3 sells 25 capes for $23, so the cost of one cape is 23/25 = 0.92 = 92 cents. We can clearly see that Store 1 has the best deal (least cost per cape), **A**.

✔️26.  We can first count the cases in which the right triangle formed lies on a face of the cube. There are 4 right triangles able to be formed per face. There are 6 faces, so for this case, there are 24 possible right triangles. Now we must count the cases in which the triangle does not lie on a face of the cube. The only type of triangle that works is one where one side is an edge of the cube, one side is the diagonal of one of the faces of the cube, and one side is a space diagonal of the cube. There are 2 right triangles per edge of the cube in this case. There are 12 edges, so for this case, there are 24 possible right triangles. Computing 24 + 24, we get 48, **C**.

✔️27.  Number each hook 1-8 from left to right. There are 4 suits, and no suit can be on 2 consecutively numbered hooks. So, the arrangements that work are (1, 3, 5, 7); (1, 3, 5, 8); (1, 3, 6, 8); (1, 4, 6, 8); and (2, 4, 6, 8). There are 5 arrangements in total. Now, we can also arrange the suits in any order from left to right, so we must multiply 5 by 4!. This gives 5 \* 24 = 120, **D**.

✔️28.  Once we find the volume of the pyramid, we can add it to the volume of the square prism and this will give us the volume of the whole tower. We know that the volume of the rectangular prism is 504, and that the square prism is as wide as possible with integer side lengths. Prime factoring 504 gives 23 \* 32 \* 7. Since the length and the width of the square prism are equal and we are trying to maximize them, they must each be 6, and the height of the square prism must be 14. The side lengths of the base of the square pyramid are also both 6. The total height of the tower is 32, so the height of just the pyramid is 32 - 14 = 18. Now we can find the volume of the pyramid, which is just ⅓ \* 6 \* 6 \* 18 = 216. Computing, 216 + 504 gives us our answer 720 units3. **B**

✔️29.  There are 33 numbers between 1 and 100 inclusive that are divisible by 3. There are 12 numbers between 1 and 100 inclusive that are divisible by 8. There are 4 numbers between 1 and 100 inclusive that are divisible by both 3 and 8, or 24. Therefore the number of numbers between 1 and 100 inclusive that are divisible by 3 or 8 but not both is 33 + 12 - 4 - 4 = 37, **B**.

✔️30.  We first number each lightbulb from 1-200, the first one being 1, the second one being 2, …, and the 200th one being 200. The total number of times a light bulb is flipped must be an odd number if it is to be on after all 200 robots pass through the room. The number of times a light bulb is flipped is equivalent to the number of factors its number label has.  If a number has an odd number of factors, it must be a perfect square. Basically, all we need to find now is the number of perfect squares from 1-200. This results in 14 being the answer, **C**.